

Quantum critical scaling and the Gross–Neveu model in $2 + 1$ dimensions

H. CHAMATI^(a) and N. S. TONCHEV^(b)

Institute of Solid State Physics, Bulgarian Academy of Sciences, 72 Tzarigradsko Chaussée, 1784 Sofia, Bulgaria

PACS 05.30.Rt – Quantum phase transitions
PACS 64.60.i – General studies of phase transitions
PACS 11.30.Qc – Spontaneous and radiative symmetry breaking

Abstract – The quantum critical behavior of the $2 + 1$ dimensional Gross–Neveu model in the vicinity of its zero temperature critical point is considered. The model is known to be renormalisable in the large N limit, which offers the possibility to obtain expressions for various thermodynamic functions in closed form. We have used the concept of finite-size scaling to extract information about the leading temperature behavior of the free energy and the mass term, defined by the fermionic condensate and determined the crossover lines in the coupling (g) – temperature (T) plane. These are given by $T \sim |g - g_c|$, where g_c denotes the critical coupling at zero temperature. According to our analysis no spontaneous symmetry breaking survives at finite temperature. We have found that the leading temperature behavior of the fermionic condensate is proportional to the temperature with the critical amplitude $\frac{\sqrt{5}}{3}\pi$. The scaling function of the singular part of the free energy is found to exhibit a maximum at $\frac{\ln 2}{2\pi}$ corresponding to one of the crossover lines. The critical amplitude of the singular part of the free energy is given by the universal number $\frac{1}{3} \left[\frac{1}{2\pi} \zeta(3) - \text{Cl}_2\left(\frac{\pi}{3}\right) \right] = -0.274543\dots$, where $\zeta(z)$ and $\text{Cl}_2(z)$ are the Riemann zeta and Clausen’s functions, respectively. Interpreted in terms the thermodynamic Casimir effect, this result implies an attractive Casimir “force”. This study is expected to be useful in shedding light on a broader class of four fermionic models.

Introduction. – Quantum phase transitions take place at zero temperature by varying some nonthermal parameter, say g , such as composition or pressure, and are driven by genuine quantum fluctuations. At rather small (when compared to characteristic excitations in the system) temperature the singularities of the thermodynamic quantities are altered. It is then expected that the leading T dependence of all thermodynamic observables is specified by the properties of the zero-temperature critical points, which take place in quantum systems. In the close vicinity of a second order quantum phase transition the coupling of statics and dynamics introduces an effective dimensionality which depends upon (imaginary) time in addition to space [1]. In this case the inverse temperature acts as a finite size in the imaginary time direction for the quantum system at its critical point. This allows the investigation of scaling laws for quantum systems near the quantum critical point in terms of the theory of finite-size

scaling [2–4].

The so called critical or thermodynamic Casimir effect, announced by Fisher and de Gennes in their study on critical binary liquids [5], is tightly related to the fluctuations of the order parameter in confined systems exhibiting temperature driven second order phase transitions in the bulk [4, 6, 7]. The named fluctuations give rise to a stress inside the confining walls, which impose effective boundary conditions on the system, depending on the behavior of the order parameter at the walls. The resulting stress can be described by the so called Casimir force, obtained as the derivative of the free energy with respect to the separation between the bounding walls. The critical Casimir force is a universal quantity, in the sense that it is independent of the details of the system. These are screened by critical fluctuations that are correlated over a long distance, namely the correlation length, that grows indefinitely as we approach the critical point. By now it is well established that the Casimir force is attractive for symmetric boundary conditions (periodic, Dirichlet, Neumann),

^(a)E-mail: chamati@issp.bas.bg

^(b)E-mail: tonchev@issp.bas.bg

while it is repulsive for non symmetric ones (antiperiodic, Dirichlet–Neumann) [4, 6, 8].

In analogy to the critical Casimir effect due to spatial confinement, we consider the critical behavior of quantum systems in the context of the critical Casimir effect, defining the temporal Casimir effect [9, 10] also dubbed *Casimir effect in time* [11], where the confinement is caused by the boundary conditions along the *finite* imaginary time direction. The singular part of the free energy of a d dimensional system in the vicinity of its quantum critical point, with hyperscaling satisfied, is expressed as

$$f_s(g, T) \approx T^{d+1} \mathfrak{Y} \left(c \mathfrak{g} T^{-1/\nu} \right). \quad (1)$$

where \mathfrak{g} and ν measure, respectively, the distance from the quantum critical point g_c and the divergence of the correlation length. The number c is a non universal quantity encoding the irrelevant details of the quantum system. The function $\mathfrak{Y}(x)$ is a universal scaling function, whose expression depends upon the universality class of the quantum critical behavior. At $g = g_c$, the universal critical amplitude $\mathfrak{Y}(0)$ is the Casimir amplitude. Its sign indicates whether the fluctuation induced “force” is attractive or repulsive. In expression (1) we assumed that the dynamic critical exponent $z = 1$. Let us mention that the critical Casimir amplitude in quantum systems is related to the specific heat and can be obtained from experimental data [4, 12].

In this letter we tackle the quantum critical scaling behavior in interacting massless fermions in order to make contact with the thermodynamic Casimir effect in time in such systems. To achieve our goal we employ the $O(2N)$ Gross–Neveu model [13] in $2 + 1$ dimensions. The model describes discrete chiral symmetry breakdown. Its Hamiltonian reads

$$\mathcal{H} = \int d^2 \mathbf{x} \left[-i \bar{\psi} \left(\gamma^1 \frac{\partial}{\partial x_1} + \gamma^2 \frac{\partial}{\partial x_2} \right) \psi - \frac{g^2}{2N} (\bar{\psi} \psi)^2 \right], \quad (2)$$

where the fields $\bar{\psi} \equiv \bar{\psi}(\mathbf{x}, t)$ and $\psi \equiv \psi(\mathbf{x}, t)$ are massless Dirac fermions with $\bar{\psi} \psi = \sum_{\alpha, i} \bar{\psi}_{\alpha}^i \psi_{\alpha}^i$, γ^{μ} are Dirac matrices and g – a nonzero coupling between fermions.

Model (2) may have applications that can be encountered in a variety of physical phenomena spanning condensed matter (superconductivity, cold matter, antiferromagnetism) and elementary particles. The model is known to be exactly solvable in the large N limit, using variational calculations [14, 15]. It can be useful in shedding light on phenomena one might expect in more realistic models.

The thermodynamics of (2) in the large N limit and at finite temperature has been investigated in many occasions (See references [14, 15] and references therein). At zero temperature it exhibits a spontaneous symmetry breaking at a critical coupling g_c . Further, investigations at finite temperature in the presence of quantum effects have shown that, under certain assumptions related to the variational problem (i.e., if only nonnegative solutions of the

corresponding self-consistency or gap equation have been taken into account), a thermal driven phase transitions takes place at $T_c > 0$. In the neighborhood of T_c , the critical behavior of a number of thermodynamics quantities has been determined.

The aim of the present study is to go beyond these works and explore the mutual influence of quantum and classical fluctuations in the $2+1$ Gross–Neveu model in the large N limit in the vicinity of g_c keeping the temperature finite ($T \rightarrow 0^+$). To this end we determine the scaling behavior of the correlation length and the free energy. In our analysis we release the constraint related to the variational problem mentioned above. We will see that this leads to a “bona fide” solution for the free energy with reasonable physical output. To the best of our knowledge this is the first time ever that such a study of the model under consideration is performed.

The remainder of this letter is organised as follows: In the next Section, we recall the main characteristics of the Gross–Neveu model and derive its critical properties in the vicinity of the quantum critical point (zero temperature critical coupling), keeping the temperature finite. To this end we compute the free energy and the gap equation to the leading order in the large N limit and analyse their corresponding behaviors as a function of the temperature. In the following Section we investigate the critical Casimir effect in time, by deriving the scaling function of the free energy density and extracting the Casimir amplitude due to the confinement in the time direction. We conclude the paper by giving a brief account of our results in the last Section.

Quantum critical behavior. – To gain insights into the thermodynamics of model (2) it is more convenient to use the field theoretic formulation with the Lagrangian density [13–15]

$$\mathcal{L}(\bar{\psi}, \psi) = i \bar{\psi} \not{\partial} \psi + \frac{g^2}{2N} (\bar{\psi} \psi)^2. \quad (3)$$

The partition function is computed from the path integral

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[- \int dt d^2 \mathbf{x} \mathcal{L}(\bar{\psi}, \psi) \right], \quad (4)$$

where the fermionic fields are antiperiodic in the imaginary time direction with period $\beta = T^{-1}$ i.e. $\psi(\mathbf{x}, \beta) = -\psi(\mathbf{x}, 0)$. Here and below we work in units such as $\hbar = k_B = 1$. The discrete chiral symmetry corresponds to

$$\psi = \gamma_5 \psi, \quad \bar{\psi} = -\bar{\psi} \gamma_5, \quad (5)$$

defining the fermionic condensate $\langle \bar{\psi} \psi \rangle$, with thermodynamic average taken with the action in eq. (4), as the order parameter or mass term of the theory. Under the above transformation the Gross–Neveu Lagrangian is invariant.

In the large N limit, instead of (3), using a standard decoupling technique [15] based on the Hubbard–Stratonovich transformation we get the equivalent effective Lagrangian:

$$\mathcal{L}_{\text{eff.}} = i\bar{\psi}\not{\partial}\psi + \lambda\bar{\psi}\psi - \frac{N\lambda^2}{2g^2}. \quad (6)$$

where $\lambda \equiv \lambda(\mathbf{x}, t)$ is an auxiliary scalar field. Let us note that in ref. [16] it was shown that the model is renormalisable order by order in $\frac{1}{N}$ expansion.

The functional integral (6) entering the expression of the partition function is Gaussian in fermionic fields. These are integrated out to yield the effective action

$$\mathcal{S}_{\text{eff.}} = \frac{N}{2g^2} \int_0^\beta dt \int d^2\mathbf{x} \lambda^2 - N \text{Tr} \ln [\not{\partial}_\mu + \lambda]. \quad (7)$$

With the aid of eq. (7) we may compute the partition function using the saddle point method. Thus we obtain an expression for the free energy to the leading order in $\frac{1}{N}$ expansion as:

$$\frac{1}{N} f(g, T) = \frac{\sigma^2}{2g^2} - \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \ln [\omega_n^2 + \mathbf{q}^2 + \sigma^2], \quad (8)$$

where $\omega_n = (2n+1)\pi T$ are the Matsubara frequencies for fermions and $\sigma \equiv \langle \lambda(\mathbf{x}, t) \rangle$ is the solution of the gap equation

$$\frac{\sigma}{g^2} = T \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{\sigma}{\omega_n^2 + \mathbf{q}^2 + \sigma^2}, \quad (9)$$

emanating from equating the derivative, with respect to σ , of the free energy $\frac{1}{N} f(g, T)$ to 0. For our further analysis we will regularise the theory by introducing a cutoff Λ over the spatial wave vector \mathbf{q} .

Eq. (9) has a trivial solution $\sigma = 0$ and a nontrivial one $\sigma \neq 0$. The stability of each solution is to be determined by comparing the corresponding free energies. Thus one can construct the phase diagram of the model under consideration.

To extract the quantum critical behavior of the model (6) at zero temperature, we write (9) in a more tractable form, employing the Schwinger representation in conjunction with the identity

$$\sum_{n=-\infty}^{\infty} e^{-(n+\frac{1}{2})^2 z} = \sqrt{\frac{\pi}{z}} \sum_{n=-\infty}^{\infty} \cos(n\pi) e^{-n^2 \frac{\pi^2}{z}} \quad (10)$$

that can be derived via the Poisson summation formula [17]. Then we have

$$\frac{1}{g^2} = \frac{1}{2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{\sqrt{\sigma^2 + \mathbf{q}^2}} + \frac{\sigma}{4\pi} - \frac{T}{2\pi} \ln 2 \cosh \frac{\sigma}{2T}, \quad (11)$$

where we have omitted the trivial solution $\sigma = 0$.

Setting $T = 0$ in (11), we find that the quantum critical point takes place at the critical coupling g_c , to be determined from

$$\frac{1}{g_c^2} = \frac{1}{2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{1}{|\mathbf{q}|}. \quad (12)$$

The quantum critical region around g_c is defined by the condition $|g - g_c| \ll 1$. From the behavior of the local zero temperature fermionic pair correlation function $\langle \bar{\psi}(\mathbf{q})\psi(-\mathbf{q}) \rangle \approx (\sigma^2 + \mathbf{q}^2)^{-1}$ at the quantum critical point, we are able to determine the critical exponent $\eta = 0$ and to identify σ^{-1} to the correlation length ξ_ψ to the leading order in the large N expansion. On the other hand the mass term, $\mathbf{m} = \langle \bar{\psi}\psi \rangle$, is given by

$$\mathbf{m} = 4\pi \left(\frac{1}{g_c^2} - \frac{1}{g^2} \right). \quad (13)$$

Hence the critical exponent $\beta = 1$ and consequently $\nu = 1$ to the leading order in $\frac{1}{N}$. Other critical exponents can be deduced via the hyperscaling relation.

Now we will turn our attention to the properties of the model in a close vicinity of the quantum critical point g_c , keeping the temperature finite. For this purpose in lieu of eq. (11), we will study the behavior of the mass term at finite temperature, say \mathbf{m}_T , using equation

$$\frac{1}{g_c^2} - \frac{1}{g^2} = \frac{T}{2\pi} \ln 2 \cosh \frac{\sigma}{2T} \quad (14)$$

obtained via dimensional regularisation. The solution of this equation is well known

$$\sigma_T = 2T \text{arccosh} \left(\frac{1}{2} e^{2\pi\kappa} \right), \quad (15)$$

where we have introduced the scaling variable

$$\kappa = \frac{1}{T} \left(\frac{1}{g_c^2} - \frac{1}{g^2} \right). \quad (16)$$

Let us note that depending on κ , σ_T can take real or complex values.

Computing the free energy with both the trivial solution, $\sigma_T = 0$, and the nontrivial one from eq. (15), we find that the phase with the nontrivial solution is stable at any $T > 0$. The free energy corresponding to this phase is shown in fig. 1. It has a maximum at T^* , which depends on the coupling g , where it coincides with that evaluated using the trivial solution. In the strong coupling region, $g > g_c$, subtracting eq. (11) from its zero temperature counterpart we obtain

$$\mathbf{m} = 2T \ln 2 \cosh \frac{\sigma}{2T}.$$

By setting $\sigma = 0$ we get

$$T^* = \frac{\mathbf{m}}{2 \ln 2}, \quad (17)$$

wherefrom we conclude that the crossover temperature T^* is proportional to the zero temperature mass term \mathbf{m} . We

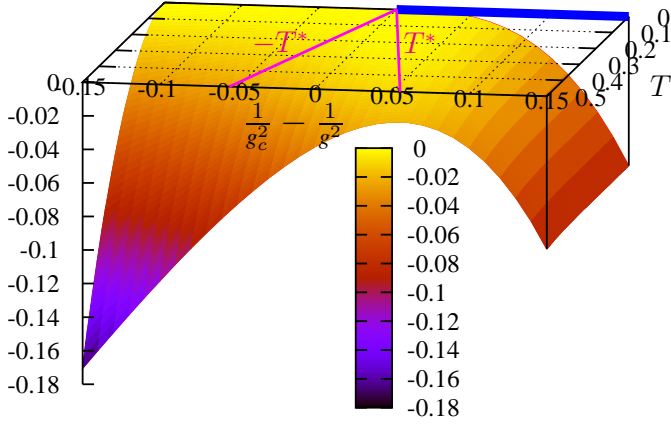


Fig. 1: The free energy as a function of the temperature and the distance from the quantum critical point. The thick line corresponds to the zero temperature ordered phase, while T^* shows a crossover line obtained by setting $\sigma_T = 0$ in eq. (14). The crossover line $-T^*$ is shown as well.

would like to notice that T^* coincides with the critical temperature of ref. [18], where the solution σ_T in (15) for the variational parameter was assumed to be positive i.e. $\sigma_T > 0$.

Now we will explore the temperature dependence of the mass term in the different regions of the (g, T) plane, including complex values for σ_T as well. In the region $\varkappa \rightarrow \infty$, by expanding the r.h.s of eq. (15) we get

$$\sigma_T = \mathfrak{m} - 2Te^{-\mathfrak{m}/T} + O(e^{-2\mathfrak{m}/T}). \quad (18)$$

This shows that in this region we have essentially the zero temperature behavior with exponentially small correction in temperature. As we approach the quantum critical point a crossover occurs at $\varkappa^* = \frac{\ln 2}{2\pi}$ (the T^* line on fig 1). For $\varkappa < \varkappa^*$ the solution σ_T is complex and seems to have no direct physical meaning. In particular so long as g approaches its critical value g_c , the solution σ_T creeps towards

$$\sigma_T^2 = -\frac{4}{9}\pi^2 T^2. \quad (19)$$

This solution shows that the relation between the fermionic correlation length and σ_T is not straightforward as in the case of zero temperature. To get a meaningful result we need to use other means, such as the asymptotic behavior of the *local* fermionic pair function. Using standard techniques [19] we end up with

$$\langle \bar{\psi}(\mathbf{q})\psi(-\mathbf{q}) \rangle \approx \frac{1}{\omega_0^2 + \sigma_T^2 + \mathbf{q}^2}. \quad (20)$$

This suggests that the thermal mass term (inverse correlation length) is defined through

$$\mathfrak{m}_T^2 = \sigma_T^2 + \pi^2 T^2, \quad (21)$$

i.e. the temperature dependent variational parameter is shifted by the lowest mode ω_0 . This is in agreement with

the fact that fermions have no zero mode at finite temperature [15]. Note that in the limit of zero temperature $\mathfrak{m} = \lim_{T \rightarrow 0} \mathfrak{m}_T$ coincides with $\sigma = \lim_{T \rightarrow 0} \sigma_T$ as expected. A similar situation is encountered in the framework of the spherical model confined in a film geometry under antiperiodic boundary conditions [8, 20].

The leading behavior of the mass term at finite temperature in the region $\varkappa \rightarrow 0^\pm$ is

$$\mathfrak{m}_T = \frac{\sqrt{5}}{3}\pi T + \frac{8}{\sqrt{15}}\pi \varkappa T + O(\varkappa^2). \quad (22)$$

By setting $g = g_c$ we obtain for the critical amplitude of the mass term at the quantum critical point the universal number $\frac{\sqrt{5}}{3}\pi$. This shows that the finite temperature mass term vanishes linearly in T as we approach the quantum critical point from above on the line $g = g_c$ in the phase diagram. Whence the dynamic critical exponent z for the model under consideration is $z = 1$.

In the limit $\varkappa \rightarrow -\infty$, we have $\sigma_T \approx i\pi T - iTe^{2\pi\varkappa}$ showing that \mathfrak{m}_T vanishes exponentially in the weak coupling region, $g < g_c$. The behavior of \mathfrak{m}_T/T as a function of \varkappa is shown in fig. 2. It is seen that the universal scaling function associated with \mathfrak{m}_T is a monotonically increasing function of the scaling variable \varkappa .

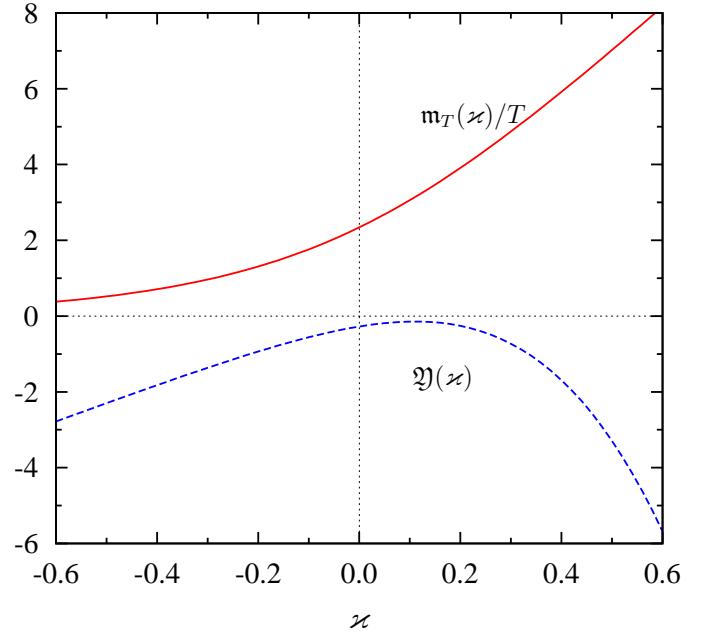


Fig. 2: Dependence of the scaling functions $\frac{\mathfrak{m}_T(\varkappa)}{T}$ and $\Psi(\varkappa)$ associated, respectively, to the finite temperature mass term (solid line) and the singular part of the free energy (dashed line) upon the scaling variable \varkappa in vicinity of the critical coupling.

To sum up, the mass term (21) normalized to the tem-

perature is found to behave as:

$$\frac{\mathbf{m}_T(\varkappa)}{T} \approx \begin{cases} \sqrt{2\pi}e^{\pi\varkappa} & \varkappa \rightarrow -\infty, \\ \frac{\sqrt{5}}{3}\pi & \varkappa = 0, \\ 4\pi\varkappa & \varkappa \rightarrow \infty, \end{cases}$$

as a function of the scaling variable \varkappa . Thus we can define three different regions in the (g, T) plane depending on the behavior of \mathbf{m}_T as a function of \varkappa . These are:

– *Renormalized classical*: The mass term diverges exponentially as we approach the zero temperature phase. For $T > 0$, thermal fluctuations destroy long-range order at any finite temperature.

– *Quantum critical*: The leading order in T of the mass term is linear.

– *Quantum disordered*: The mass term is independent of the temperature.

These regions are separated by the crossover lines T^* and $-T^*$ on fig 1. In fig. 3 we show the qualitative crossover diagram of model (2). It is worth noticing that similar behaviors were found in studies involving many other quantum systems (see e.g. ref. [3] and references therein).

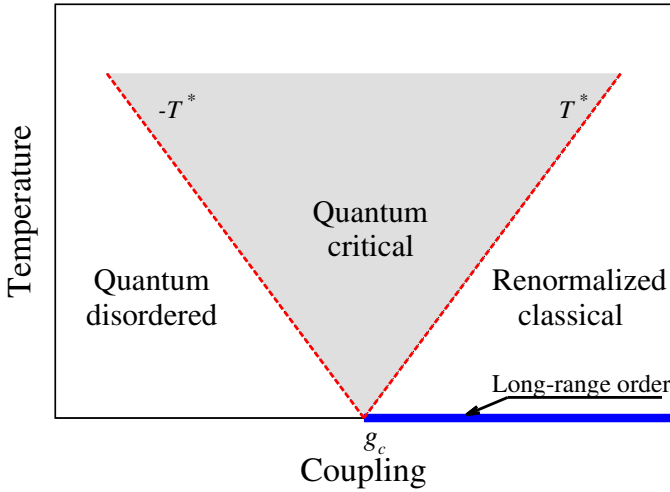


Fig. 3: Qualitative crossover diagram of model (2) in the quantum critical region. The thick line shows the zero temperature ordered phase, while the dashed lines are the crossover lines of fig 1.

Scaling behavior of the free energy. – In the quantum critical region, $|g - g_c| \ll 1$, using the representation

$$\ln z = \int_0^\infty \frac{dt}{t} (e^{-t} - e^{-zt}), \quad z > 0, \quad (23)$$

and dimensional regularisation, the singular part of the well known expression for the free energy (8) can be written in the scaling form

$$\frac{1}{N} f_s(g, T) = T^3 \mathfrak{Y}(\varkappa), \quad (24)$$

where we have introduced the universal scaling function

$$\mathfrak{Y}(\varkappa) = -\frac{1}{2}\varkappa \left(\frac{\sigma_T}{T}\right)^2 + \frac{1}{12\pi} \left(\frac{\sigma_T}{T}\right)^3 + \frac{1}{2\pi} \left[\frac{\sigma_T}{T} \text{Li}_2\left(-e^{-\frac{\sigma_T}{T}}\right) + \text{Li}_3\left(-e^{-\frac{\sigma_T}{T}}\right) \right], \quad (25)$$

in order to make contact with eq. (1). Here $\sigma_T \equiv \sigma_T(\varkappa)$ is the solution (15) of the gap equation (9) and

$$\text{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}, \quad |z| < 1,$$

are polylogarithms [21].

Let us now explore the behavior of the scaling function $\mathfrak{Y}(\varkappa)$. Away from the quantum critical point the leading behavior is given by

$$\mathfrak{Y}(\varkappa) \approx \begin{cases} \frac{1}{2} \left[\pi^2 \varkappa + \frac{1}{\pi} \zeta(3) \right] & \varkappa \rightarrow -\infty, \\ -\frac{8}{3} \pi^2 \varkappa^3 & \varkappa \rightarrow \infty. \end{cases}$$

This shows that the scaling function is linear in \varkappa in the limit $\varkappa \rightarrow -\infty$ and decreases as $-\varkappa^3$ in the opposite limit.

In fig. 2 we show the behavior of the universal scaling function $\mathfrak{Y}(\varkappa)$. It has a maximum estimated to be

$$\mathfrak{Y}(\varkappa^*) = \frac{f_s(g, T^*)}{(T^*)^3} = -\frac{3}{8\pi} \zeta(3) = -0.143485...,$$

at $\varkappa = \varkappa^* = \frac{\ln 2}{2\pi} = 0.110318...$, corresponding to the crossover line T^* of fig. 1.

At the quantum critical point g_c we may obtain an analytic expression using the relations [21]

$$\text{Li}_3(e^{i\frac{\pi}{3}}) = \frac{1}{3} \zeta(3) + i \frac{5\pi^3}{162} \quad (26)$$

and

$$\text{Li}_2(e^{i\theta}) = \left[\frac{\pi^2}{6} - \frac{\theta}{4}(2\pi - \theta) + i \text{Cl}_2(\theta) \right]; \quad 0 \leq \theta \leq 2\pi, \quad (27)$$

where Clausen’s function [22] is defined through

$$\text{Cl}_2(\theta) = \sum_{n=1}^{\infty} \frac{\sin(n\theta)}{n^2}.$$

Thus the critical amplitude of the singular part of the free energy of the massless Gross–Neveu model is given by

$$\begin{aligned} \mathfrak{Y}(0) &= \left[\frac{1}{6\pi} \zeta(3) - \frac{1}{3} \text{Cl}_2\left(\frac{\pi}{3}\right) \right] \\ &= -0.274543... \end{aligned} \quad (28)$$

It is negative. In terms of the Casimir effect this implies an attractive Casimir “force”, despite the fact that the boundary conditions imposed over the imaginary time are antiperiodic. A similar attractive behavior for the Casimir force was found in reference [23, 24] induced by the vacuum fluctuations of fermionic fields between two parallel plates.

Discussion. – We have investigated the quantum critical behavior of the massless Gross–Neveu model, as a prototype for four–fermionic models. At infinite N it has a property that is rarely encountered in the realm of critical phenomena, namely exact solvability. This property allows the derivation of thermodynamic functions in closed form, without recourse to perturbative methods. We have investigated the critical behavior of the model in the vicinity of the quantum critical point (zero temperature critical coupling) and the changes caused by switching on the temperature. In particular we have paid attention to the interpretation of the quantum critical scaling in terms of the thermodynamic Casimir effect, resulting from the confinement in the imaginary time direction.

Notice that the present investigation rules out any phase transition at finite temperature. Such a transition was obtained in previous studies assuming that the solution (15) of the gap equation obeyed by the variational parameter of the effective theory can take only positive real values. Here such a constraint is released and we consider complex values as well. This is justified by the fact that the free energy is always real as shown in figs. 1 and 2. In this case it was found appropriate to redefine the mass term through shifting the variational parameter by the lowest fermionic mode.

The behavior of the free energy and the mass term in the vicinity of the quantum critical point was investigated in details. We have identified the three different regions typical in quantum critical phenomena: Quantum disordered, quantum critical and renormalized classical (see fig. 3). These are separated by crossover lines corresponding to $|g - g_c| \sim T$. In particular, we would like to emphasize that it was found the mass term is linear in T in the region $\frac{1}{T}|g - g_c| \rightarrow 0$. The critical amplitude of the mass term, which coincides with the inverse of the correlation length describing fermionic correlations, was determined to be $\frac{\sqrt{5}}{3}\pi$.

The temperature dependence of the singular part of the free energy in the vicinity of the quantum critical point is investigated in details. Its associated scaling function is found to exhibit a maximum at $\frac{\ln 2}{2\pi}$. The corresponding critical amplitude may be interpreted as the Casimir amplitude, due to the confinement in the time direction. This is computed exactly [see eq. (28)] and found to be negative as it is the case for the quantum nonlinear sigma model [25], although this last model obeys the Bose–Einstein statistics. This shows that the confinement in the imaginary time direction is reflected in different way on the sign of the Casimir amplitude in comparison with the confinement in a space direction.

This work was supported by the Bulgarian Fund for Scientific Research Grant No. TK-X-1712/2007 (N.T.).

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